Lab 7: Numerical Analysis

INSERT YOUR NAME HERE (INSERT YOUR UW NETID HERE)

Due by 23:59pm on Mar 8, 2024

Total Points: 30

Part 1. Basic Root-Finding Problems (8 pts)

Use both the bisection method and fixed-point iteration to find an approximation to $\sqrt[3]{25}$ that is accurate to within 10^{-7}.

1. For the bisection method, use $f(x) = x^3 - 25$ with the search interval $[1, 3]$.
2. For the fixed-point iteration, use $g(x) = \frac{2x^3+25}{3x^2}$ to identify the fixed point $g(x) = x$ with some initial point (says, $p_0 = 1$ or $p_0 = 2$).

```
# Your code goes here
```

Part 2. Non-quadratic Convergence of Newton's Method (10 pts)

Recall that Newton's method does not exhibit quadratic convergence when one of the following two cases occur:

- *Case 1*: The derivative of the function f is zero at the root p with $f(p) = 0$.
- *Case 2*: The second-order derivative of the function f does not exist at the root p with $f(p) = 0$.

We already demonstrate the exact linear convergence of Newton's method under *Case 1* in the Lecture 8 slides. Now, we will explore *Case 2* by implementing Newton's method on $f(x) = x + x^{\frac{4}{3}}$. In addition, output the limiting point p^* of Newton's method under the initialization $p_0 = 3$ and the tolerance level $\epsilon = 10^{-13}$. Moreover, apply the Aitken's Δ^2 method to the sequence produced by Newton's method. Finally, plot the **logarithm** of the error $|p_n - p^*|$ against the number of iterations for both the Newton's method and Aitken's Δ^2 method in the same plot. (Hint: You can adopt the code in Lecture 8 slides, but the `ylim` and legend location for the plot need adjusting.)

```
library(latex2exp)
# Your code goes here
```

Part 3. Numerical Differentiation and Integration (6+6 pts)

1. Compute the first-order derivative of $f(x) = \log(x) + cos(x) - \sqrt{x}$ at $x = 2$ using the forward-difference, three-point endpoint formula, and five-point midpoint formula with $h = 0.005$. Then, also compute its second-order derivative at $x = 2$ using the second-order derivative midpoint formula. Output all these values. Finally, also output $f'(2)$ and $f''(2)$ that are computed by hand with R as your calculator.

```
# Your code goes here
```

2. Compute the integral $\int_0^{\pi} \exp(2\cos(x))dx$ using the trapezoidal rule, Simpson's rule, composite Simpson's rule with $n = 60$, and composite Trapezoidal rule with $n = 30$. Output all these values. In addition, output the integral value using the build-in function `integrate()`.

```
# Your code goes here
```