

Your title

Your name*

June 19, 2018

Abstract

A concise summary of the whole reading report and main findings

1 Introduction

A brief overview of the subject or research field that you are going to discuss.

2 Background

Definition 2.1. A list of vectors $x_1, \dots, x_k \in \mathbf{C}^n$ is orthogonal if $x_i^* x_j = 0$ for all $i \neq j, i, j \in \{1, \dots, k\}$. If, in addition, $x_i^* x_i = 1$ for all $i = 1, \dots, k$ (that is, the vectors are normalized), then the list is orthonormal.

Example 2.2 (normalization). If $y_1, \dots, y_k \in \mathbf{C}^n$ are orthogonal and nonzero, the vectors x_1, \dots, x_k defined by $x_i = (y_i^* y_i)^{-\frac{1}{2}} y_i, i = 1, \dots, k$ are orthonormal.

Theorem 2.3. Every orthogonal list of vectors in \mathbf{C}^n is linearly independent.

Proof. Suppose that $\{y_1, \dots, y_k\}$ is an orthogonal set. Normalize them as Example 2.2 did and obtain an orthonormal list of vectors $\{x_1, \dots, x_k\}$. Assume that $0 = \alpha_1 x_1 + \dots + \alpha_k x_k$. Then $0 = (\alpha_1 x_1 + \dots + \alpha_k x_k)^* (\alpha_1 x_1 + \dots + \alpha_k x_k) = \sum_{i,j} \bar{\alpha}_i \alpha_j x_i^* x_j =$

$\sum_{i=1}^k |\alpha_i|^2 x_i^* x_i = \sum_{i=1}^k |\alpha_i|^2$ because the vectors x_i are orthonormal. Thus, all $\alpha_i = 0$ and hence $\{x_1, \dots, x_k\}$ is a linearly independent set, which in turn means that $\{y_1, \dots, y_k\}$ is linearly independent. \square

3 Method and Evaluation

Main section of the reading report. It contains your solutions to the exercises, your methodology to address the problem, and your experimental results.

*Your institution

4 Conclusions

Summarize what you have written in previous sections and discuss your understandings of possible future research directions.

References

- [1] Roger A. Horn, Charles R. Johnson (2012) *Matrix Analysis, Second Edition*. Cambridge University Press.